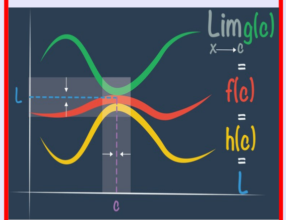
Math 261

Spring 2021

Lecture 45



Class QZ 15

Evaluate

1) $\int_{-1}^{1} x^{100}\,dx = \left.\frac{x^{101}}{101}\right|_{x=-1}^{x=1}$

$= \left(\frac{1^{101}}{101} - \frac{(-1)^{101}}{101}\right) = \boxed{\frac{2}{101}}$

2) $\int_{1}^{8} x^{-2/3}\,dx$

$= \left.\frac{x^{-2/3+1}}{-2/3+1}\right|_{1}^{8} = \left.\frac{x^{1/3}}{1/3}\right|_{1}^{8} = \left.3\sqrt[3]{x}\right|_{1}^{8}$

$= 3\left(\sqrt[3]{8} - \sqrt[3]{1}\right) = 3(2-1) = \boxed{3}$

3) $\int_{0}^{\frac{\pi}{4}} \sec x \tan x\,dx$

$= \left.\sec x\right|_{x=0}^{x=\frac{\pi}{4}}$

$= \sec\frac{\pi}{4} - \sec 0$

$= \boxed{\sqrt{2} - 1}$

Fundamental Theorem for Calculus Part I:

If f is continuous on $[a, b]$, then

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x) \quad \text{for all } a \le x \le b$$

1) Find $\frac{d}{dx}\int_1^x \frac{t}{1+t^3}\,dt = \frac{x}{1+x^3}$

2) Find $\frac{d}{dx}\int_2^x \frac{t^2}{1+\cos^2(t^3)}\,dt = \frac{x^2}{1+\cos^2(x^3)}$

Now $\frac{d}{dx}\int_{u(x)}^{v(x)} f(t)\,dt = f(v(x))\cdot v'(x) - f(u(x))\cdot u'(x)$

Find $\frac{d}{dx}\int_{x^2}^{x^3} \sin^3 t^2\,dt = \sin^3(x^3)^2\cdot 3x^2 - \sin^3(x^2)^2\cdot 2x$

$$= \boxed{3x^2\sin^3 x^6 - 2x\sin^3 x^4}$$

Find $\frac{d}{dx}\int_{\sqrt{x}}^{t^2} \cos t^2 \, dt$

$v(x)$

$u(x)$ $f(t)$ $v'(x)$ $u'(x)$

$= \cos(x^2)^2 \cdot (2x) - \cos(\sqrt{x})^2 \cdot \frac{1}{2\sqrt{x}}$

$= 2x \cos x^4 - \frac{\cos x}{2\sqrt{x}}$

Given $f(x) = \int_0^x (1-t^2)\cos^2 t \, dt$

Find the interval where $f(x)$ is increasing. where $f'(x) > 0$

$f'(x) = \frac{d}{dx}\int_0^x (1-t^2)\cos^2 t \, dt$

$= (1-x^2)\cos^2 x \cdot 1 - 0$

$f'(x) = (1-x^2)\cos^2 x$ ≥ 0

$1-x^2 > 0$ $(1+x)(1-x) > 0$

$-$ $+$ $-$
-1 1

$f'(x) > 0$ on $(-1, 1)$

$(-1, 1)$

Function $f(x)$ is increasing where $f'(x) > 0$

Discuss the Concavity for the Function

$$f(x) = \int_0^x \frac{t^2}{t^2+t+2}\,dt.$$

$t^2+t+2 \to$ Never Zero

we need $f''(x)$

$$f'(x) = \frac{x^2}{x^2+x+2}\cdot 1 - \frac{0^2}{0^2+0+2}\cdot 0$$

$$f'(x) = \frac{x^2}{x^2+x+2}$$

$$f''(x) = \frac{2x(x^2+x+2) - x^2(2x+1)}{(x^2+x+2)^2}$$

$$= \frac{\cancel{2x^3} + 2x^2 + 4x - \cancel{2x^3} - x^2}{(x^2+x+2)^2} = \frac{x^2+4x}{(x^2+x+2)^2}$$

$$= \frac{x(x+4)}{(\quad)^2}$$

$f''(x) = 0$ at $x=0, x=-4$

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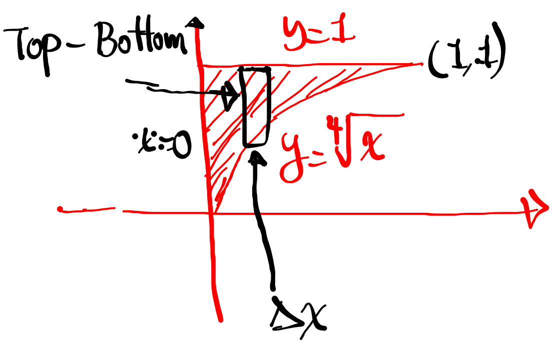
at $x=-4$ $x=0$

Find the shaded area below



Method I

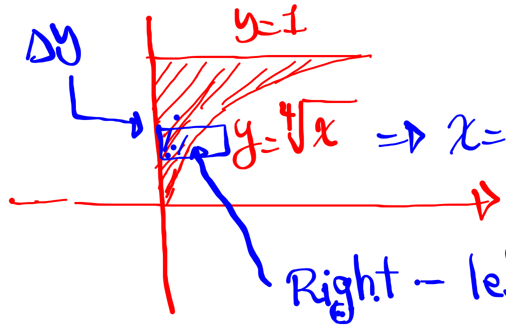
$x^{1/4}$

$$A = \int_0^1 \left[1 - \sqrt[4]{x}\,\right] dx$$

$$= \left(x - \frac{x^{5/4}}{5/4}\right)\Bigg|_{x=0}^{x=1}$$

$$= 1 - \frac{4}{5} = \boxed{\frac{1}{5}}$$

Find the shaded area below



$\Rightarrow x = y^4$

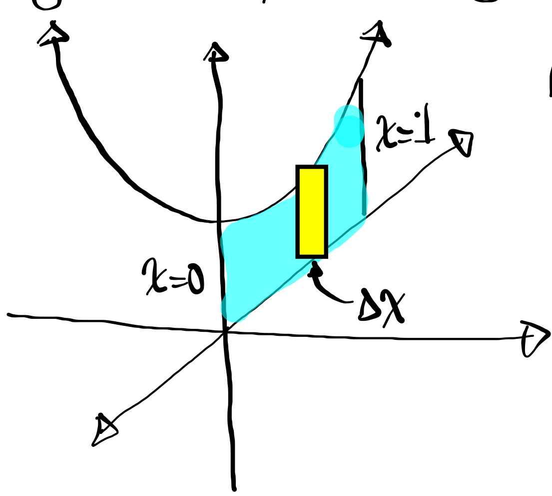
Right − left

$y^4 - 0$

$$A = \int_0^1 (y^4 - 0)\,dy$$

$$= \frac{y^5}{5}\Big|_0^1 = \boxed{\frac{1}{5}}$$

Find the area enclosed by $x=0$, $x=1$, $y = x^2+1$, and $y = x$.



$$A = \int_0^1 \Big[\overbrace{x^2+1}^{\text{Top}} - \underbrace{x}_{\text{Bottom}}\Big]\,dx$$

$$= \left(\frac{x^3}{3} + x - \frac{x^2}{2}\right)\Bigg|_{x=0}^{x=1}$$

$$= \frac{1}{3} + 1 - \frac{1}{2} = \frac{2+6-3}{6} = \boxed{\frac{5}{6}}$$

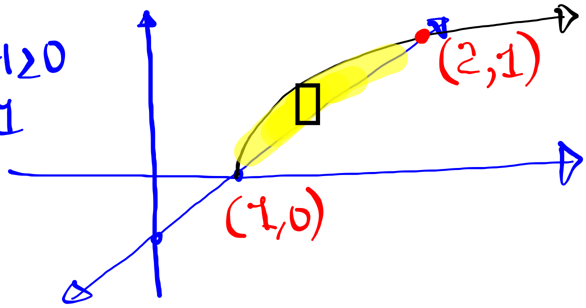
Find the area enclosed by

$y = \sqrt{x-1}$ and $x - y = 1$.

x	y
0	-1
1	0

$x - 1 \geq 0$

$x \geq 1$



$y = \sqrt{x-1}$

$y = x - 1$

$\sqrt{x-1} = x - 1$

$x - 1 = (x-1)^2$

$x = 2$

$A = \int_1^2 \left[\sqrt{x-1} - (x-1)\right] dx$

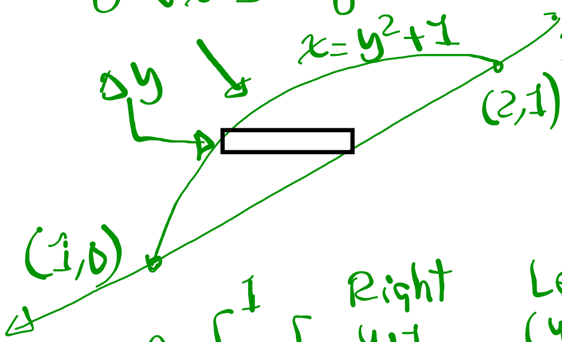
1) Given $\int_2^8 f(x)\,dx = 10$, find $\int_8^2 f(x)\,dx$ $= \boxed{-10}$

2) Find $\int_5^5 \sin^2(x^3)\,dx = \boxed{0}$

$y = \sqrt{x-1}$ $\quad y^2 = x-1$

$x = y^2+1$ $\quad x - y = 1$

$x = y+1$



Right $\quad$ Left

$$A = \int_0^1 \left[y+1 - (y^2+1) \right] dy$$

$$= \int_0^1 \left[y - y^2 \right] dy = \left(\frac{y^2}{2} - \frac{y^3}{3} \right) \Big|_0^1$$

$$= \frac{1}{2} - \frac{1}{3} = \boxed{\frac{1}{6}}$$